# Towards Combining Probabilistic, Interval, Fuzzy Uncertainty, and Constraints: An Example Using the Inverse Problem in Geophysics

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Combining uncertainty models within cyberinfrastructure is a challenging problem. The main objective of cyberinfrastructure is to be able to seamlessly move data between different databases (where this data is stored in different formats), to feed the combined data into a remotely located program (which may require yet another data format), and to return the result to the user. It is also important to gauge the quality and accuracy of this result. We may have different models for describing uncertainty of different databases and programs; it is therefore important to be able to combine different uncertainty models.

<u>A case study</u>: *The seismic inverse problem.* The existing techniques for solving the seismic inverse problem, such as Hole's tomographic inversion code [2], only take into account the measurement results. Expert geophysicists also have some prior knowledge about the structure of the region. If we could take this prior knowledge into account when processing the geophysical measurements, this would greatly improve the accuracy of the geophysical structures uncovered by solving the inverse problem and also aid the inversion to converge.

*Probabilistic prior knowledge*. Measurement results often come with probabilistic uncertainty (i.e., we know the probabilities of different values of measurement errors). As a result of processing such measurement results, we usually obtain not only the estimate of the desired quantity, but also the standard deviation (and other statistical characteristics) of possible differences between the actual and estimated values of the desired quantity. Some prior knowledge comes from previous measurement processing, and has therefore the form \the value $x$ is approximately equal to $x'$, with standard deviation $\sigma$. This prior knowledge has been successfully used; see, e.g., [3].

*The need to use interval and fuzzy prior knowledge*. In some situations, we have a different type of prior knowledge (e.g., a geophysicist may know that the speed of sound at a certain depth must be between 6 and 8 km/s). We could, in principle, describe this information in probabilistic terms by assuming that 7 is the most probable value and 1 is the standard deviation, but this would be a distorted representation of the expert knowledge because, in contrast to this representation, the expert does not necessarily consider any value within the interval [6,8] to be more probable than others. Often, instead of a fixed interval, the expert can produce several nested intervals corresponding to different degrees of certainty, in effect, a fuzzy number. In [1], we describe techniques for handling such interval and fuzzy uncertainty.

*The need to combine different types of uncertainty*. In real life, some prior knowledge comes from prior data processing, some from prior interval constraints. In this presentation, we describe techniques that allow us to combine different types of uncertainty. The ultimate goal is to provide visual presentations of the combined effects of different types of uncertainty.

<u>References</u>

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